Journal of Applied Mechanics and Technical Physics, Vol. 45, No. 4, pp. 542–545, 2004

MOTION OF A SPHERE IN A LIQUID CAUSED BY VIBRATIONS OF ANOTHER SPHERE

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UDC 532.582

The motion of an absolutely rigid sphere in a nonuniformly vibrating, ideal, incompressible liquid is considered. The liquid vibrates under the action of an absolutely rigid sphere vibrating in a specified manner. Refined conditions are obtained under which the inclusion sphere recedes from the vibrating sphere, approaches it, and does not perform mean motion. It is found that under nonuniform vibrations of the liquid, the motion of the inclusion can depend on the geometrical parameters of the hydromechanical system.

Key words: *liquid, inclusion, uniform and nonuniform vibrations of a liquid.*

1. Vibrational action on a liquid with inclusions can lead to the effects of a mean, monotonic motion of the inclusions [1–14]. This circumstance can be used to control inclusions in liquids [5, 10, 15, 16]. Of special interest is the problem of controlling inclusions whose density coincides with the liquid density. This problem is related to the possibility of dividing liquid vibrations into uniform and nonuniform [17] (also see [16]).

The problem of the motion of an absolutely rigid spherical inclusion in a nonuniformly vibrating ideal, incompressible liquid acted upon by an absolutely rigid sphere vibrating in a specified manner was formulated and solved in [3]. This study revealed that an inclusion whose density is lower than the liquid density recedes from the vibrating body and an inclusion whose density is higher than the liquid density approaches the vibrating body. It was found [3] that in the approximation considered, an inclusion whose density coincides with the liquid density does not perform mean motion (is at rest on the average). Thus, the study of [3] posed the question of whether liquid vibrations can cause the mean motion of an inclusion if the density of the inclusion coincides with the liquid density? A positive answer to this question was given in [18] for the case where liquid vibrations are caused by a doublet that has a time-varying moment. However, in view of the applied value of problems in this research area, it is important to have theoretical results that would effectively motivate directed experimental studies. Such results should precisely indicate (if possible) experimental conditions under which one might expect a manifestation of the effects found theoretically. As applied to the question posed by the study of [3], this implies that theoretical results should contain a parameter of the dimension of length that characterizes the vibrating body (real vibrating bodies are extended). In view of the aforesaid, we studied [14] the problem of the motion of an absolutely rigid inclusion in the form of a circular cylinder in a nonuniformly vibrating, ideal, incompressible liquid acted upon by an absolutely rigid circular cylinder vibrating in a specified manner. It was found [14] that if the inclusion density coincides with the liquid density, the inclusion performs mean motion and approaches the vibrating body (which is in accordance with the findings of [18]). The formulas of [14] that define the mean motion of inclusions contain the radius of the vibrating body.

In the present paper, we study the motion of an absolutely rigid spherical inclusion in a nonuniformly vibrating, ideal, incompressible liquid. The liquid is subjected to the vibrational action from an absolutely rigid sphere vibrating in a specified manner. The formulation of the problem is different from that in [3]. A higher approximation than in [3] is implemented. It is found that if the inclusion density coincides with the liquid density,

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Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 45, No. 4, pp. 102–106, July–August, 2004. Original article submitted October 7, 2003.

the inclusion approaches the vibrating body; refined conditions were obtained under which the inclusion recedes from the vibrating body, approaches the vibrating body, and does not perform mean motion.

2. In an unbounded, incompressible, ideal liquid there are two absolutely rigid spheres. At the initial time t (at $t = 0$), the liquid and the spheres are at rest about the inertial rectangular coordinate system x, y, z; the centers of the spheres are on the y axis. At subsequent times, the first sphere of radius a_1 (a vibrating body) performs specified periodic translational vibrations along the y axis with period T; the second sphere of radius a_2 (an inclusion) performs translational motion along the y axis under the action of the liquid pressure forces; the liquid flow is potential and axisymmetric. The position of the first sphere is defined by the radius-vector

$$\boldsymbol{H} = H\boldsymbol{e}_y$$

of the center of the first sphere [$\boldsymbol{e}_y = (0, 1, 0)$ and $H = A(1 - \cos(2\pi t/T)$, where A is a constant]. The position of the second sphere is defined by the radius-vector

$$\boldsymbol{S} = S\boldsymbol{e}_y \tag{1}$$

of the center of the second sphere ($S > H + a_1 + a_2$). It is required to establish how the second sphere moves.

We assume that S_0 is the value of S at $t = 0$, (q_1) and (q_2) are the surfaces of the first and second spheres, respectively, $\boldsymbol{n}_1$ is the normal to (q_1), $\boldsymbol{n}_2$ is the outward unit normal to (q_2), Φ is the liquid velocity potential, P is the liquid pressure,

$$F = -\iint\limits_{(q_2)} P\boldsymbol{n}_2 \cdot \boldsymbol{e}_y \, dq_2 \tag{2}$$

is the force acting in the y direction on the second sphere from the liquid, ρ_{sphere} is the density of the second sphere, ρ_{liq} is the liquid density, and I is an arbitrary function of t.

The coordinate S, pressure P, and potential Φ satisfy the following equations and conditions:

$$\frac{4}{3}\pi a_2^3 \rho_{\text{sphere}} \frac{d^2S}{dt^2} = F; \tag{3}$$

$$S = S_0, \quad \frac{dS}{dt} = 0 \qquad \text{at} \quad t = 0; \tag{4}$$

$$\frac{\partial\Phi}{\partial t} + \frac{1}{2}(\nabla\Phi)^2 + \frac{P}{\rho_{\text{liq}}} = I; \tag{5}$$

$$\Delta\Phi = 0; \tag{6}$$

$$\boldsymbol{n}_1 \cdot \nabla\Phi = \boldsymbol{n}_1 \cdot \boldsymbol{e}_y \frac{dH}{dt} \qquad \text{in} \quad (q_1); \tag{7}$$

$$\boldsymbol{n}_2 \cdot \nabla\Phi = \boldsymbol{n}_2 \cdot \boldsymbol{e}_y \frac{dS}{dt} \qquad \text{in} \quad (q_2); \tag{8}$$

$$\nabla\Phi \to 0 \quad \text{at} \quad x^2 + y^2 + z^2 \to \infty. \tag{9}$$

3. We consider problem (3)–(9) for small values of $\alpha = a_2/S_0$ compared with unity (value $\beta = a_1/a_2$ and $\gamma = A/a_2$ are not small or large compared with unity).

Using the method of determining the liquid velocity potential described in [19], we obtain the solution of problem (6)–(9) that satisfies Eqs. (6) and (9) exactly and Eqs. (7) and (8) approximately, with accuracy up to quantities proportional to dH/dt and dS/dt and small compared with $\alpha^9\, dH/dt$ and $\alpha^9\, dS/dt$, respectively. Using (2) and (5) and the indicated solution of problem (6)–(9), we obtain

$$F = \frac{\pi a_2^4 \rho_{\text{liq}}}{T^2}\Big\{\frac{2\alpha^3\beta^3}{(s-\alpha h)^3}\Big[1 + \frac{\alpha^6\beta^3}{(s-\alpha h)^6}\Big]\frac{d^2h}{d\tau^2} + \frac{6\alpha^4\beta^3}{(s-\alpha h)^4}\Big[1 - \frac{\alpha^3\beta^3}{(s-\alpha h)^3} - \frac{4\alpha^5\beta^3}{(s-\alpha h)^5}\Big]\Big(\frac{dh}{d\tau}\Big)^2 - \frac{12\alpha^6\beta^3}{(s-\alpha h)^7}$$
$$\times\Big[1 + \frac{4\alpha^2\beta^2}{(s-\alpha h)^2}\Big]\frac{dh}{d\tau}\frac{ds}{d\tau} - \frac{1}{\alpha}\Big[\frac{2}{3} + \frac{2\alpha^6\beta^3}{(s-\alpha h)^6} + \frac{6\alpha^8\beta^5}{(s-\alpha h)^8}\Big]\frac{d^2s}{d\tau^2} + \frac{6\alpha^5\beta^3}{(s-\alpha h)^7}\Big[1 + \frac{4\alpha^2\beta^2}{(s-\alpha h)^2}\Big]\Big(\frac{ds}{d\tau}\Big)^2\Big\}, \tag{10}$$

where $\tau = t/T$, $h = H/a_2 = \gamma(1 - \cos 2\pi\tau)$, and $s = S/S_0$.

Let us assume that for $\alpha \to 0$ and constants τ, β, γ, and λ

$$s \sim s_0 + \alpha s_1 + \ldots + \alpha^{10} s_{10}. \tag{11}$$

According to (3), (4), (10), and (11), in the zeroth, first, ..., and tenth approximations, we have problems for $s_0, s_1, \ldots, s_{10}$. Solving the indicated problems, we obtain

$$s_0 = 1, \qquad s_1 = 0, \qquad s_2 = 0, \qquad s_3 = 0,$$

$$s_4 = s_4', \qquad s_5 = s_5', \qquad s_6 = s_6', \qquad s_7 = s_7',$$

$$s_8 = 3\beta^6 \lambda(\lambda - 1) \int\limits_0^\tau d\tau' \int\limits_0^{\tau'} \Big(\frac{dh}{d\tau''}\Big)^2 d\tau'' + s_8', \tag{12}$$

$$s_9 = 21\beta^6 \lambda(\lambda - 1) \int\limits_0^\tau d\tau' \int\limits_0^{\tau'} h\Big(\frac{dh}{d\tau''}\Big)^2 d\tau'' + s_9',$$

$$s_{10} = 84\beta^6 \lambda(\lambda - 1) \int\limits_0^\tau d\tau' \int\limits_0^{\tau'} h^2\Big(\frac{dh}{d\tau''}\Big)^2 d\tau'' - 12\beta^6 \lambda \int\limits_0^\tau d\tau' \int\limits_0^{\tau'} \Big(\frac{dh}{d\tau''}\Big)^2 d\tau'' + s_{10}',$$

where $\lambda = 3\rho_{\text{lig}}/(2\rho_{\text{sphere}} + \rho_{\text{liq}})$ and $s_4', s_5', \ldots, s_{10}'$ are periodic functions of τ. Using (11) and (12), we obtain

$$s = \hat{s} + s'; \tag{13}$$

$$s = \hat{s} + 21\pi^2 \alpha^9 \beta^6 \gamma^3 [\lambda - 1 + 5\alpha\gamma(\lambda - 1 - 4/(35\gamma^2))]\tau^2 + s'', \tag{14}$$

s' and s'' are periodic functions of τ.

4. Relations (1), (13), and (14) approximately define the dependence of $\boldsymbol{S}$ on t [(14 has higher accuracy compared with (13)]. The second sphere, moving along the y axis, performs vibrations and mean monotonic motion.

Expression (15) for $\hat{s}$ coincides with the corresponding expression for Y/Y_0 resulting from formulas (22) and (24) of [3]. According to (13), for $\rho_{\text{sphere}} < \rho_{\text{liq}}$, the second sphere recedes from the first sphere, and for $\rho_{\text{sphere}} > \rho_{\text{liq}}$, it approaches the first sphere; the second sphere is at rest on the average if

$$\rho_{\text{sphere}} = \rho_{\text{liq}}. \tag{16}$$

Formula (14), in particular, demonstrates the refined result concerning the behavior of the second sphere at $\rho_{\text{sphere}} = \rho_{\text{liq}}$: the second sphere is not at rest and approaches the first sphere.

5. Let us consider the question of under what condition the second sphere does not perform mean motion.

We assume that for $\alpha \to 0$, the constants τ, β, and γ, and $\rho_{\text{sphere}}/\rho_{\text{liq}} = 1 + a\alpha + b\alpha^2$ (a and b are constants), we have

$$s \sim \sigma_0 + \alpha\sigma_1 + \ldots + \alpha^{10}\sigma_{10}. \tag{17}$$

According to (3), (4), (10), and (17) in the zero, first ..., tenth approximations, we have problems for $\sigma_0, \sigma_1, \ldots, \sigma_{10}$, respectively. Solving the indicated problems, we obtain

$$\sigma_0 = 1, \qquad \sigma_1 = 0, \qquad \sigma_2 = 0, \qquad \sigma_3 = 0, \qquad \sigma_4 = \sigma_4',$$

$$\sigma_5 = \sigma_5', \qquad \sigma_6 = \sigma_6', \qquad \sigma_7 = \sigma_7', \qquad \sigma_8 = \sigma_8',$$

$$\sigma_9 = -2\beta^6 a \int\limits_0^\tau d\tau' \int\limits_0^{\tau'} \Big(\frac{dh}{d\tau''}\Big)^2 d\tau'' + \sigma_9', \tag{18}$$

$$\sigma_{10} = -2\beta^6(2a^2 - b - 6) \int\limits_0^\tau d\tau' \int\limits_0^{\tau'} \Big(\frac{dh}{d\tau''}\Big)^2 d\tau'' - 14a\beta^6 \int\limits_0^\tau d\tau' \int\limits_0^{\tau'} h\Big(\frac{dh}{d\tau''}\Big)^2 d\tau'' + \sigma_{10}',$$

where $\sigma'_4, \sigma'_5, \ldots, \sigma'_{10}$ are periodic functions τ. From (17) and (18), it follows that the second sphere is at rest on the average if

$$\rho_{\rm sphere} = (1 - 6\alpha^2)\rho_{\rm liq} \tag{19}$$

[(19) has higher accuracy than (16)].

We note that (19) can also be obtained from (14).

6. The state of rest determined by relation (19) separates the other two states of qualitatively different motion of the second sphere. If $\rho_{\rm sphere} \neq (1 - 6\alpha^2)\rho_{\rm liq}$, the second sphere recedes from the first sphere for $\rho_{\rm sphere} < (1 - 6\alpha^2)\rho_{\rm liq}$ and approaches it for $\rho_{\rm sphere} > (1 - 6\alpha^2)\rho_{\rm liq}$. If $\rho_{\rm sphere} < \rho_{\rm liq}$, the second sphere recedes from the first sphere for $\alpha < \sqrt{(\rho_{\rm liq} - \rho_{\rm sphere})/(6\rho_{\rm liq})}$ and approaches it for $\alpha > \sqrt{(\rho_{\rm liq} - \rho_{\rm sphere})/(6\rho_{\rm liq})}$. This shows that under nonuniform vibrations of the liquid, the change in the nature of motion of the inclusion can depend on the geometrical parameters of the hydromechanical system.

The results imply new possibilities of controlling inclusions in liquids.

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